B-math 2nd year Final Exam/Back paper
Subject : Analysis

Time : 3.00 hours Max.Marks 75.

1. Use Stokes theorem to evaluate the line integral

$$\int_C -y^3 dx \; + x^3 \; dy - z^3 \; dz$$

where C is the intersection of the cylinder $x^2+y^2 = 3$ and the plane $x+y+z = 1$. (15)

2.Evaluate the double integral

$$\int\int_{\Omega} \frac{x}{\sqrt{16+y^7}} dxdy$$

where the set Ω is bounded above by the line $y = 2$, below by the graph of $y = x^{\frac{1}{3}}$ and on the left by the y-axis. (10)

3. Use the divergence theorem to evaluate

$$\int\int\int_V div(\vec{F}) \; dxdydz$$

where V is the solid cylinder $\{(x, y, z) : x^2 + y^2 < 1, 0 < z < 1\}$ and

$$\vec{F}(x, y, z) := (1 - (x^2 + y^2)^3)\mathbf{i} + (1 - (x^2 + y^2)^3)\mathbf{j} + (x^2z^2)\mathbf{k}.$$

(15)

4. Consider the system of equations

$$\begin{aligned} 2e^{x_1} + x_2y_1 - 4y_2 + 3 &= 0 \\ x_2cos(x_1) - 6x_1 + 2y_1 - y_3 &= 0 \end{aligned}$$

a) Show that there exists (x_1, x_2) such that the point $(x_1, x_2, 3, 2, 7) \in I\!R^5$ satisfies the above equation.

b) Is it possible to obtain x_1 and x_2 as differentiable functions of (y_1, y_2, y_3) in a neighborhood of $(3, 2, 7)$ such that $(x_1(y_1, y_2, y_3), x_2(y_1, y_2, y_3), y_1, y_2, y_3)$ satisfy the above equations ? Justify your answer. (2+3)

5. a) If (f_n) and (g_n) converge uniformly on a set E prove that $(f_n + g_n)$ converges uniformly on E.
b) Prove that the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$$

converges uniformly in every bounded interval, but does not converge absolutely for any value of x. (7+8)

6. Let $\vec{r} : T \to S$ be a smooth parametrization of a surface S in $I\!R^3$ over a domain T in $I\!R^2$. Let $E(u, v) := \partial_u \vec{r} \cdot \partial_u \vec{r} = \|\partial_u \vec{r}\|^2$; $F(u, v) := \partial_u \vec{r} \cdot \partial_v \vec{r}$; $G(u, v) := \partial_v \vec{r} \cdot \partial_v \vec{r} = \|\partial_v \vec{r}\|^2$. Show that

$$Area\ of\ S = \int\int_T \sqrt{EG - F^2}\, dudv.$$

You may need the result that for vectors $\vec{a}$ and $\vec{b}$, $\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\||sin(\delta)|$ where δ is the angle between $\vec{a}$ and $\vec{b}$. (15)